

Finding b -colorings using feedback edges

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A b -coloring of a graph is a proper vertex coloring such that each color class contains a vertex that sees all other colors in its neighborhood. This type of coloring was introduced by Irving and Manlove [1]. They proved that the b -COLORING problem, in which the task is to decide whether a graph admits a b -coloring with k colors, is NP-complete in general but solvable in polynomial time on trees. Moreover, it is known that b -COLORING is in XP when parameterized by clique-width [2] (and hence also by tree-width) but W[t]-hard for all $t \in \mathbb{N}$ when parameterized by path-width [3] (and hence also by tree-width).

We study a parameterization that is more restrictive than tree-width but still captures trees. Namely, we show that b -COLORING is fixed-parameter tractable when parameterized by the feedback edge number, which is the number of edges one needs to delete to obtain a forest. Our algorithm combines standard techniques used in parameterized algorithmics with the problem-specific ideas used in the polytime algorithm for trees. In addition, we present an FPT algorithm for b -COLORING parameterized by distance to co-cluster, which is the number of vertices one needs to delete to obtain a complete multipartite graph. Finally, we observe that b -COLORING is W[1]-hard when parameterized by tree-depth and FPT when parameterized by vertex integrity and carving-width (and hence also by cut-width).

REFERENCES

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