Density of traceable graphs

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We consider the minimum number of edges of traceable graphs, i.e. graphs that have a Hamiltonian path, for graphs that have a specific structure. Clearly, if we do not impose any additional restrictions, the minimum number of edges of an n-vertex traceable graph is $n - 1$. If we restrict our attention to traceable graphs which have additional properties, e.g. bounded neighborhood diversity, we obtain a larger bound on the number of edges. More precisely, we consider several structural graph parameters and ask the following question: *What is the minimum number of edges an n-vertex graph has to have if it is traceable and has a bounded parameter d?*

We show the following tight upper and lower bounds:

- quadratic for the class of graphs of bounded neighborhood diversity, bounded size of maximum induced matching, or bounded cluster vertex deletion number;
- $n \log n$ for the class of cographs or, more generaly, bounded modular-width, and for the class of bounded distance to cograph; and
- sligthly superlinear for the class of bounded shrub-depth.