

Girth-regular graphs of degree 4 and small girth

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(joint work with Maruša Lekše and Primož Potočnik)

For a graph Γ of finite girth, each vertex v of Γ can be associated with a list of integers, one integer for each edge incident with v , representing the number of girth cycles containing the edge. If the lists associated with the vertices of Γ are all identical (and so Γ is regular), we say that Γ is *girth-regular*, and the shared list (ordered in a non-decreasing way) is said to be the *signature of Γ* . Additionally, if all the values in the list are the same, we say that Γ is *edge-girth-regular*.

The concept of girth-regularity was introduced in [1] as a generalization of vertex-transitivity, with applications mainly in classifying cubic vertex-transitive graphs of small girth [1, 2, 3]. In this talk, we discuss girth-regular graphs of higher degree and present an “almost” classification of girth-regular graphs of degree 4 and girth 3, and edge-girth-regular graphs of degree 4 and girth 4.

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