On minimum Venn diagrams and dual problems

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(joint work with Sofia Brenner, Torsten Mütze, and Francesco Verciani)

An Venn diagram with n curves is called *simple* if in every crossing only two curves intersect. Simple Venn diagrams have exactly $2^n - 2$ crossings. On the other hand, the number of crossings in any n-Venn diagram is at least $L_n := \lceil \frac{2^n-2}{n-1} \rceil$, and Bultena and Ruskey conjectured that (non-simple) Venn diagrams with this minimal number of crossings exist for every n. We establish an asymptotic version of this conjecture by constructing n-Venn diagrams with $(1 + \frac{33}{8n})L_n$ crossings when n is a power of 2, which extends to $(2 + o(1))L_n$ for any n by a doubling trick [1]. These (geometric) constructions are based on (combinatorial) partitions of the hypercube into isometric paths. I will also give an overview of other recent advances on Venn diagrams obtained by considering hypercube problems in the dual graphs and list some open problems.

References

[1] S. Brenner, P. Gregor, T. Mütze, F. Verciani, On minimum Venn diagrams, 42nd International Symposium on Computational Geometry (SoCG 2026), LIPIcs 367 (2026) to appear.