Various algebraic constructions of regular (di)graphs

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(Parts of this work are joint with Robert Jajcay.)

Constructions of graphs based on algebraic structures play an important role in modern graph theory, as they often allow one to translate questions about graph invariants into problems concerning operations within the underlying algebraic structure. This algebraic perspective can significantly simplify both the analysis and the construction of graphs with prescribed properties.

In addition to well-established methods such as Cayley graphs and constructions based on voltage assignments (lifts), the talk will also present approaches that make use of quasigroups, loops, Latin squares, group cosets, and equitable partitions. These frameworks provide flexible and powerful tools for generating not only highly symmetric graphs, but also graphs that are less symmetric in the terms of number of automorphisms.

We will further demonstrate how these constructions can be applied in extremal graph theory, particularly in the search for edge-girth-regular graphs, as well as in the study of directed strongly regular graphs, where algebraic methods often lead to new examples and deeper structural insights.

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