Lower bound on the maximum denominator of fractional chromatic numbers

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(joint work with Marthe Bonamy, Tomáš Kaiser, and Jean-Sébastien Sereni)

Let $\mathrm{sd}(n)$ denote the least positive integer d such that for every n-vertex graph G there exist integers p and $q \leq d$ such that its fractional chromatic number is $\frac{p}{q}$. An upper bound on the determinants of Hadamard matrices implies that $\mathrm{sd}(n) \leq 2^{-n}(n+1)^{(n+1)/2}$ [1]. Erdős had raised the question of whether every n-vertex graph G has $\chi_f(G) = \frac{p}{q}$, with p, q positive integers and $q \leq n$. Chvátal, Garey and Johnson [2] gave a negative answer by presenting a construction (based on successive joins of odd cycles with suitable lengths) which yields a lower bound on $\mathrm{sd}(n)$ that is super-polynomial in n. So far, the best known lower bound obtained using an iterated Mycielski construction is asymptotically about $1.346^n/\sqrt{\log n}$ [3].

Since the upper bound is superexponential whereas the lower bound is only exponential, the gap between them is enormous. We present a new family of graphs that allows us to prove that $\mathrm{sd}(n) \geq 2^{n/2}$.

References

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