

Measuring depth of matroids

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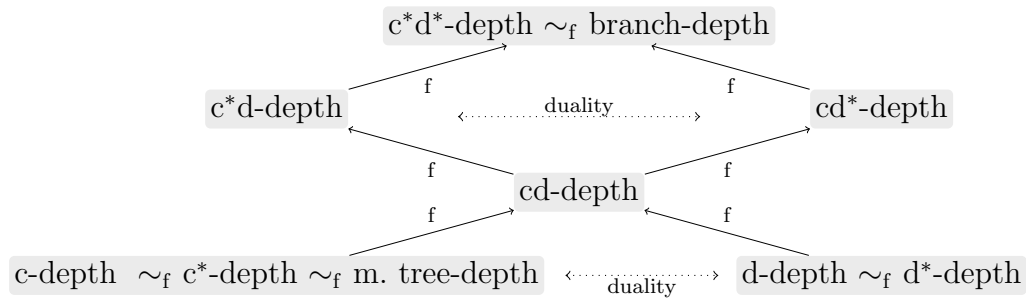
(joint work with Jakub Balabán, Petr Hliněný, and Kristýna Pekárková)

Recently, connections between matroid depth measures and block-structured integer programming have been discovered [2, 3], motivating a systematic study of recursive depth parameters for matrices and matroids. In this talk, we discuss a unifying framework [1] of matrix and matroid depth measures and their relation to graph depth measures.

Recall that tree-depth is defined by removing a vertex, and then recursing on connected components. Our framework takes a similar approach; it measures depth by simplifying the matrix or matroid using elementary operations, and then recursing on connected components.

We consider the following two pairs of dual elementary operations: *c-transformation* contracts an element of the matroid or a column of the matrix; *c*-transformation* adds¹ a new element (or column) e and contracts e ; *d-transformation* is dual to *c-transformation*; and *d*-transformation* is dual to *c*-transformation*. These define 8 depth measures: *c*-, *d*-, *cd*-, *c**-, *d**-, *c*d*-, *cd**-, and *c*d**-depth.

We [1] have proven the equivalence of the new definitions with existing depth measures. Notably, the *c*-, *d*-, *cd*-, *c**-, *d**-, *c*d*-, and *cd**-depths of a matrix A are equal to the corresponding depths of its vector matroid $M(A)$. The hierarchy of functionally equivalent matroid depth measures is depicted below:



REFERENCES

- [1] J. Balabán, P. Hliněný, J. Jedelský, K. Pekárková, Measuring depth of matroids, arXiv:2604.04896, 2026.
- [2] M. Brianski, M. Kouřtecký, D. Král, K. Pekárková, F. Schröder, Characterization of matrices with bounded graver bases and depth parameters and applications to integer programming, 49th International Colloquium on Automata, Languages, and Programming (ICALP 2022), LIPIcs 229 (2022) 29:1–29:20.

¹Note that we can only add an element in a way that creates a matroid.

- [3] T. F. N. Chan, J. W. Cooper, M. Kouřtecký, D. Král, K. Pekárková, Matrices of optimal tree-depth and row-invariant parameterized algorithm for integer programming, 47th International Colloquium on Automata, Languages, and Programming (ICALP 2020), LIPIcs 168 (2020) 26:1–26:19.