Balanced biregular cages

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(joint work with Gabriela Araujo-Pardo)

The cage problem is a classical problem in extremal graph theory. A (k, g)-graph is a k-regular graph with girth g. A (k, g)-*cage* is a (k, g)-graph of minimum order. In this talk we consider a generalizations of this problem.

An $(r, s; g)$-*balanced biregular graph* (for short a babi-graph) is a graph of girth g having degree set equal to $\{r, s\}$ and satisfying the additional property that the number of vertices of degree r equals to the number of vertices of degree s. An $(r, s; g)$-*balanced biregular cage* (for short an $(r, s; g)$-babi-cage) is an $(r, s; g)$-babi-graph of minimum order. We give general lower and upper bounds on the size of babi-graphs and present the babi-cages for $g = 3$ and 4. We construct relatively small babi-graphs with girth 5 and 6 from incidence graphs of finite projective, affine and biaffine planes and show that some of them are babi-cages.

References

[1] G. Araujo-Pardo, Gy. Kiss, On balanced biregular cages, arXiv:2604.22395, 2026.