Census of Cayley-likeness of small vertex-transitive graphs

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The notion that helps us understand global symmetry of the graph Γ is its graph automorphisms, which form a group under composition - $Aut(\Gamma)$. Interpreting "Γ being symmetric" as the fact that "it looks the same, no matter which vertex we are inspecting Γ from" means that $Aut(\Gamma)$ acts transitively on Γ's vertices. The property of vertex-transitivity is actively researched in theoretical graph theory, enumerated by computers [1], while still having practical applications.

Transitive graphs contain many interesting graphs, which includes the whole class of Cayley graphs - constructed from groups and their generating sets. As it turns out, Cayley graphs are a proper subset of vertex-transitive graphs and the well-known Petersen graph is the smallest example of a vertex-transitive graph that is not a Cayley graph. Such graphs are known as "non-Cayley" graphs. Our aim is to more granularly divide the class of vertex-transitive according to "how close to Cayley" each graph is. This work can be viewed as a continuation of research in [2] as well as [3].

In this talk, we combine their approaches with the computational power to classify vertex-transitive graphs of order $n \leq 30$ into a hierarchy with respect to the notions of *deficiency* introduced in [3].

References

[1] D. Holt, G. Royle, A census of small transitive groups and vertex-transitive graphs, J. Symbolic Comput. 101 (2020) 51–60.

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[3] R. Jajcay, G. A. Jones, r-regular families of graph automorphisms, European J. Combin. 79 (2019) 97–110.