Regular difference distance magic oriented graphs

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(joint work with Jiří Havlíček)

Let G be an oriented graph. The weight of vertex x in a vertex labeling f of G is defined as the sum of the vertex labels directed into x minus the sum of vertex labels directed out of x, that is

$$wt(x) = \sum_{y \in N^+(x)} f(y) - \sum_{y \in N^-(x)} f(y).$$

A difference distance magic (DDM) labeling of an oriented graph G is a bijection $f : V(G) \to \{1, 2, \ldots, |V(G)|\}$ where there exists $k \in N$ such that for all $x \in V(G)$ the vertex weight is $wt(x) = k$. That is, $wt(x)$ is the same integer k for all vertices in G, which we will refer to as the magic constant.

The labeling was introduced by Altman et al. in 2024. In this talk we present a classification of orders for which such DDM orientation of an r-regular graph exists: for $r = 3$ we give a full classification of even orders and a partial classification for odd orders, for $r = 4$ we give a full classification of all feasible orders. Moreover, we present constructions of connected 4-regular graphs that allow a DDM orientation and some general results on DDM orientations of k-regular graphs.

References

[1] K. Altman, L. Calzado, B. Freyberg, P. Kovář, E. Lewis, A. Marr, L. Ross, R. Via, Difference distance magic oriented graphs, Res. Math. Sci. 11 (2024) 67.