Hypercube drawings with no long plane paths

Anna Margarethe Limbach

(joint work with Todor Antić, Niloufar Fuladi, and Pavel Valtr)

We study the existence of plane substructures in drawings of the d-dimensional hypercube graph Q_d. We construct drawings of Q_d which contain no plane subgraph with more than $2d-2$ edges, no plane path with more than $2d-3$ edges, and no plane matching of size more than $2d-4$. On the other hand, we prove that every rectilinear drawing of Q_d with vertices in convex position contains a plane path of length d (if d is odd) or $d-1$ (if d is even). We also prove that if a graph G is a plane subgraph of every drawing of Q_d for a sufficiently large d, then G is necessarily a forest of caterpillars. Lastly, we give a short proof of a generalization of a result by Alpert et al. [1] on the maximum rectilinear crossing number of Q_d.

References

[1] M. Alpert, E. Feder, H. Harborth, S. Klein, The maximum rectilinear crossing number of the n dimensional cube graph, Congr. Numer. 195 (2009) 147–158.