On the local structure of uniquely 3-colorable plane graphs

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A graph G is called uniquely k-colorable if $\chi(G) = k$ and every k-coloring of G induces the same partition of the vertex set of G into k independent sets. It is known that uniquely 3-colorable plane graphs possess interesting structural properties: except of C_3, they contain at least three triangles ([1]) and in the case they contain exactly four triangles, two of them are adjacent ([3]). Moreover, they contain a 3-face adjacent to a face of size at most 5 ([2]).

In addition to these results, we show that every uniquely 3-colorable plane graph of minimum degree at least 3 contains an edge of type $(3, \leq 9), (4, \leq 6)$ or $(5, 5)$ (the bounds being sharp); a similar result holds for 3-vertex paths (with all vertices having degrees at most 13), but not for longer paths. Also, extending the result of [2], we prove that every uniquely 3-colorable plane graph contains a cluster of three faces of sizes at most 7 as well as a cluster of four faces of size at most 9 (this bound being sharp).

References

[1] V.A. Aksionov, On uniquely 3-colorable planar graphs, Discrete Math. 20 (1977) 209–216.

[2] Z. Li, N. Matsumoto, E. Zhu, J. Xu, T. Jensen, On uniquely 3-colorable plane graphs without adjacent faces of prescribed degrees, Mathematics 7(9) (2019) 793.

[3] Z. Li, E. Zhu, Z. Shao, J. Xu, A note on uniquely 3-colourable planar graphs, Int. J. Comput. Math. 94(5) (2017) 1028–1035.