S-packing colorings of distance graphs with distance sets of cardinality 2

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(joint work with Boštjan Brešar, Jasmina Ferme, Přemysl Holub, and Marko Jakovac)

For a non-decreasing sequence $S = (s_1, s_2, \dots)$ of positive integers, a partition of the vertex set of a graph G into subsets $X_1, \dots, X_\ell$, such that vertices in X_i are pairwise at distance greater than s_i for every $i \in \{1, \dots, \ell\}$, is called an S-packing ℓ-coloring of G. The minimum ℓ for which G admits an S-packing ℓ-coloring is called the S-packing chromatic number of G and is denoted by $\chi_S(G)$. In this talk, S-packing colorings of distance graphs $G(\mathbb{Z}, \{k, t\})$ are considered. Given positive integers k and t, the distance graph $G(\mathbb{Z}, \{k, t\})$ is the infinite graph whose vertex set is $\mathbb{Z}$, and two vertices $x, y \in \mathbb{Z}$ are adjacent whenever $|x - y| \in \{k, t\}$. In the presented paper [2], existing partial results [1, 3, 4] were complemented, thus determining all values of $\chi_S(G(\mathbb{Z}, \{k, t\}))$ when S is any sequence with $s_i \leq 2$ for all i. In particular, if $S = (1, 1, 2, 2, \dots)$, then the S-packing chromatic number is 2 if $k + t$ is even, and 4 otherwise, while if $S = (1, 2, 2, \dots)$, then the S-packing chromatic number is 5, unless $\{k, t\} = \{2, 3\}$ when it is 6; when $S = (2, 2, 2, \dots)$, the corresponding formula is more complex.

References

[1] B. Benmedjdoub, I. Bouchemakh, É. Sopena, 2-distance colorings of integer distance graphs, Discuss. Math. Graph Theory 39(2) (2019) 589–603.

[2] B. Brešar, J. Ferme, P. Holub, M. Jakovac, P. Melicharová, S-packing colorings of distance graphs with distance sets of cardinality 2, Appl. Math. Comput. 490 (2025) 129200.

[3] B. Brešar, J. Ferme, K. Kamenická, S-packing colorings of distance graphs $G(\mathbb{Z}, \{2, t\})$, Discrete Appl. Math. 298 (2021) 143–154.

[4] P. Holub, J. Hofman, On S-packing colourings of distance graphs $D(1, t)$ and $D(1, 2, t)$, Appl. Math. Comput. 447 (2023) 127855.