Tiling the sphere with regular polygons

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(joint work with Hoi Ping Luk and Christopher Purcell)

We give a complete classification of edge-to-edge tilings of the sphere by regular polygons under a unified framework. Without assuming convexity of the tiles or polyhedrality of the underlying graph, our proof is independent of the Johnson-Zalgaller classification of solids with regular faces (1967) [1, 2], which took over 200 pages. We apply a blend of trigonometric, algebraic and combinatorial tools of independent interest. Our main theorem follows.

Theorem 1. *The edge-to-edge spherical tilings by regular polygons are*

- *the five Platonic tilings,*
- *the thirteen Archimedean tilings,*
- *the twenty-five tilings corresponding to circumscribable Johnson-Zalgaller solids:*

$$J_1, J_2, J_3, J_4, J_5, J_6, J_{11}, J_{19}, J_{27}, J_{34}, J_{37}, J_{62}, J_{63},$$
$$J_{72}, J_{73}, J_{74}, J_{75}, J_{76}, J_{77}, J_{78}, J_{79}, J_{80}, J_{81}, J_{82}, J_{83},$$

- *the infinite families of prisms and antiprisms,*
- *the infinite families of hosohedra and dihedra.*

References

[1] V. A. Zalgaller, Convex polyhedra with regular faces, Zap. Naučn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. (LOMI) 2 (1967) 220 pp.

[2] N. W. Johnson, Convex polyhedra with regular faces, Canad. J. Math. 18 (1966) 169–200.