# Local irregularity problems for multigraphs

**Alfréd Onderko**

(joint work with Anna Flaszczyńska and Igor Grzelec)

A (multi)graph $G$ is locally irregular (l.i. in short) if adjacent vertices have different degrees in $G$. Clearly, not every graph is l.i., but it may admit a decomposition into l.i. graphs. In [1] it was proved that every connected graph except specific subcubic cacti admits a decomposition into a finite number of l.i. graphs. While it is conjectured that every l.i. decomposable graph admits a decomposition into at most four l.i. subgraphs, the best general upper bound is 220, proved in [6].

In [2, 3, 4, 5], the extensions of this problem to multigraphs were considered. With respect to these results, we conjecture that there is a constant $k_0$ such that every $k$-multigraph ${}^kG$ (that is, every multiedge has multiplicity $k$), except ${}^kK_2$, is decomposable into at most two l.i. multigraphs, for every $k \geq k_0$. In this talk, we provide several results on this conjecture, utilizing the connection between l.i. decompositions and different neighbor-distinguishing colorings. In particular, we determine such a constant $k_0$ if $G$ contains a spanning subgraph $H$ with $1 \leq \delta(H) \leq \Delta(H) \leq 2$. This yields a corollary for graphs $G$ with $\Delta(G) \leq 2\delta(G)$. We also discuss possible ways to overcome the current obstacles to prove the conjecture.

## References

[1] O. Baudon, J. Bensmail, J. Przybyło, M. Woźniak, On decomposing regular graphs into locally irregular subgraphs, European. J. Combin. 49 (2015) 90–104.

[2] I. Grzelec, T. Madaras, A. Onderko, R. Soták, On a new problem about the local irregularity of graphs, Appl. Math. Comput. 512 (2026) 129763.

[3] I. Grzelec, A. Onderko, M. Woźniak, On local irregularity conjecture for 2-multigraphs, Appl. Math. Comput. 514 (2026) 129832.

[4] I. Grzelec, M. Woźniak, On decomposing multigraphs into locally irregular submultigraphs, Appl. Math. Comput. 452 (2023) 128049.

[5] I. Grzelec, M. Woźniak, Local irregularity conjecture for 2-multigraphs versus cacti, Opuscula Math. 44(1) (2024) 49–65.

[6] B. Lužar, J. Przybyło, R. Soták, New bounds for locally irregular chromatic index of bipartite and subcubic graphs, J. Comb. Optim. 36(4) (2018) 1425–1438.