Random embeddings of graphs

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A random 2-cell embedding of a connected graph G in some orientable surface is obtained by choosing a random local rotation around each vertex. Under this setup, the number of faces or the genus of the corresponding 2-cell embedding becomes a random variable. Random embeddings of a few particular graph classes have been well-understood. However, the general case remains elusive.

In his breakthrough work ([1] and a series of other papers), Stahl developed the foundation of "random topological graph theory". Most of his results have been unsurpassed until recently.

It was recently shown [2] that for any graph G, the expected number of faces is at most linear. In [3] it was shown that for most graphs the expected number is much smaller, logarithmic or polylogarithmic.

In this talk we will summarize recent progress in this on-going project.

References

[1] S. Stahl, Permutation-partition pairs: A combinatorial generalization of graph embeddings, Trans. Amer. Math. Soc. 259(1) (1980) 129–145.

[2] J. C. Loth, B. Mohar, Expected number of faces in a random embedding of any graph is at most linear, Combin. Probab. Comput. 32(4) (2023) 682–690.

[3] J. C. Loth, K. Halasz, T. Masařík, B. Mohar, R. Šámal, Random embeddings of graphs: the expected number of faces in most graphs is logarithmic, Proceedings of the 2024 Annual ACM-SIAM Symposium on Discrete Algorithms (SODA) 1177–1193.