Cycle covers of cubic graphs with small colouring defect

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A long-standing conjecture of Alon and Tarsi (1985) states that every 2-connected graph can have its edges covered with cycles of total length at most $7/5 \cdot m$, where m is the number of edges. As with several related conjectures, the case of cubic graphs is crucial for understanding the conjecture.

In this talk we deal with the relationship between the minimum length of a cycle cover and the colouring defect of a cubic graph. This invariant, introduced in 2015 by Steffen [1], is defined as the minimum number of edges left uncovered by any set of three perfect matchings of a cubic graph. We show that every 2-connected cubic graph with colouring defect not exceeding 3 admits a cycle cover of length at most $4/3 \cdot m + 1$, just one step above the universal lower bound of $4/3 \cdot m$ for all cubic graphs. We also prove that, regardless of defect, the same bound holds for 2-connected cubic graphs that have an edge whose end-vertices removed yield a 3-edge-colourable graph and the edge lies on a 5-cycle. Finally, we introduce a new invariant for cubic graphs, their covering excess, which measures the deviation of the length of a shortest cycle cover from the mentioned universal lower bound mentioned above. We show that every 2-connected cubic graph with covering excess at most 1 admits a cycle double cover.

References

[1] E. Steffen, 1-Factor and cycle covers of cubic graphs, J. Graph Theory 78 (2015) 195–206.