Colorings of plane graphs with small facial defect

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(joint work with Igor Fabrici)

Two adjacent edges of a plane graph are *facially adjacent* if they are consecutive in the cyclic order around their end vertex (given by the embedding). An edge coloring is *facially proper* if any two facially adjacent edges obtain different colors. Similarly, a total coloring is facially proper if any two adjacent vertices, any two facially adjacent edges and any two incident elements obtain different colors. Jendroľ [1] proved that each plane graph has a vertex 3-coloring such that, on each face, any monochromatic path consists of at most two vertices, i.e. each color has facial vertex defect at most two (we denote such a coloring as $[2, 2, 2]$-vertex coloring).

In this talk, we show that each plane graph has $[1, 2, 2]$-edge coloring (there are no facially adjacent edges colored by 1 and, on each face, monochromatic paths colored by 2 or 3 consist of at most two edges) and $[1, 1, 1, 2, 2]$-total coloring.

References

[1] S. Jendroľ, On a 3-coloring of plane graphs without monochromatic facial 3-paths, Discuss. Math. Graph Theory 46(2) (2026) 341–351.